What you'll learn about

- Derivatives of Inverse Functions
- Derivatives of the Arcsine
- Derivatives of the Arctangent
- Derivatives of the Arcsecant
- Derivatives of the Other Three

… and why

The relationship between the graph of a function and its inverse allows us to see the relationship between their derivatives.

EQ: How do we apply the derivatives of the inverse trig functions?





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## Theorem 3: Derivatives of Inverse Functions

If $f$ is differentiable at every point of an interval $I$ and $\frac{dy}{dx}$ is never zero on $I$, then $f$ has an inverse and $f^{-1}$ is differentiable at every point on the interval $f(I)$.





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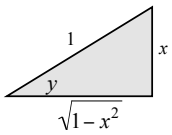
## Derivative of the Arcsine

Suppose we have the function $y = \sin^{-1} x$

$y = \sin^{-1} x \Rightarrow x = \sin y$

Differentiate implicitly: $1 = \cos y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

Chain Rule — See Diagram Below




In General:

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

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## Derivative of the Arcsine

If $u$ is a differentiable function of $x$ with $|u|<1$, we apply the Chain Rule to get

$$\frac{d}{dx}\sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx},\ |u|<1.$$

Why is it necessary for $|u|<1$???




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## Example **Derivative of the Arcsine**

If $y=\sin^{-1}8x^2$, find $\frac{dy}{dx}$.

$y=\sin^{-1}8x^2$ where $u=8x^2$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-\left(8x^2\right)^2}}\left(\frac{du}{dx}\left(8x^2\right)\right)$$
$$=\frac{1}{\sqrt{1-64x^4}}(16x)$$
$$=\frac{16x}{\sqrt{1-64x^4}}$$




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## Derivative of the Arctangent

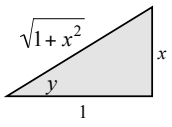
Suppose we have the function $y=\tan^{-1}x$

$y=\tan^{-1}x \Rightarrow x=\tan y$

Differentiate implicitly: $1=\sec^2 y\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{\sec^2 y}=\frac{1}{1+x^2}$

Chain Rule

See Diagram Below




In General:

$$\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$$

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## Derivative of the Arctangent

The derivative is defined for all real numbers.

If $u$ is a differentiable function of $x$, we apply the Chain Rule to get

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}.$$





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## Example **Derivative of the Arctangent**

If $y = \tan^{-1} 3x$, find $\dfrac{dy}{dx}$

$y = \tan^{-1} 3x$ where $u = 3x$

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\left(\frac{du}{dx}(3x)\right) = \frac{3}{1+9x^2}$$





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## Derivative of the Arcsecant

Suppose we have the function $y = \sec^{-1} x$

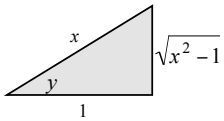
$y = \sec^{-1} x \Rightarrow x = \sec y$

Differentiate implicitly:

$$1 = \sec y \tan y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\sec y \tan y} = \frac{1}{|x|\sqrt{x^2-1}}$$

Chain Rule

See Diagram Below




In General:

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

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## Derivative of the Arcsecant

If $u$ is a differentiable function of $x$ with $|u| > 1$, we have the formula

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2 - 1}}\frac{du}{dx}, \quad |u| > 1.$$




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## Example **Derivative of the Arcsecant**

Given $y = \sec^{-1}(3x - 4)$, find $\frac{dy}{dx}$.

$y = \sec^{-1}(3x - 4)$ where $u = 3x - 4$

$$\frac{dy}{dx} = \frac{1}{|3x-4|\sqrt{(3x-4)^2 - 1}}\frac{d}{dx}(3x - 4)$$

$$= \boxed{\frac{3}{|3x-4|\sqrt{9x^2 - 24x + 15}}}$$




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## Inverse Function – Inverse Cofunction Identities

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$\csc^{-1} x = \frac{\pi}{2} - \sec^{-1} x$$

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## Calculator Conversion Identities

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$$





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## Example Derivative of the Arccotangent

Find the derivative of $y = \cot^{-1} x^2$.

Rewrite $y = \cot^{-1} x^2$ as $y = \frac{\pi}{2} - \tan^{-1} x^2$ where $u = x^2$.

$$\frac{dy}{dx} = 0 - \frac{1}{1 + \left(x^2\right)^2}\left(2x\right)$$

$$= -\frac{2x}{1 + x^4}$$





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## Assignment: Page 175 #1 – 25 odds